Lecture 11: Closed Box and Beyond

The Closed Box Model

“Closed Box” model with constant yield:

\[ Z(t) = -\rho \ln(\mu(t)) \]

Metallicity: \( \mu \)
Gas fraction: \( \rho \)

But (with infall): \( Z(t) \rightarrow \rho \) as \( \mu \rightarrow 0 \)

\[ Yield = \rho = \frac{\text{mass of new metals added to ISM by SNe}}{\text{mass of ISM converted to long-lived stars}} = \frac{(Z_{\text{SN}} - Z)(1-\alpha)}{\alpha} \]

Closed Box model ignores:
1. IGM–ISM exchanges: IGM falls in, ISM blown out of galaxy
2. SN Ia, stellar winds, PNe, novae, etc.
3. Initial enrichment by e.g. Pop.III stars prior to galaxy formation?
4. Faster enrichment (more SNe) in denser regions of galaxy.

The G-dwarf problem

: Expected number of metal-poor stars in SN

Mass of stars with \( Z=0.25Z_0 \) compared to the mass in stars with the current metallicity of gas:

\[ \frac{M_*(<0.25Z_0)}{M_*(<0.7Z_0)} = \frac{[1-e^{-0.25Z_0/\alpha}]}{[1-e^{-0.7Z_0/\alpha}]} \approx 0.54 \]

Expect ~50% of stars near Sun to have \( Z<0.25Z_0 \)

However only ~2% of F-G dwarf stars in SN have \( Z>0.25Z_0 \)

G-dwarf problem
G-dwarf problem: Possible solutions

1. Super-enriched local ISM (i.e., statistical fluke)
2. System started with higher metallicity $Z_0 = 0.15Z_\odot$
3. Close box not valid, i.e.,
   1. Gas lost through winds, i.e., leaky-box model
   2. Gas accreted, i.e., accreting-box model

Leaky Box model

Winds of massive stars (AGB) and SN can eject gaseous material out of a galaxy.

As this gas is heavily enriched it can lead to the loss of metals from the ISM
The leaky-box model

Let us assume an outflow of processed material given by \( g(t) \), then:

\[
\frac{\partial M_*}{\partial t} = (Z - \rho) \frac{\partial M_*}{\partial t} - Z g
\]

Conservation of mass gives us:

\[
\frac{\partial M_g}{\partial t} + \frac{\partial M_*}{\partial t} + g = 0
\]

Let's adopt an outflow rate proportional to the star-formation rate:

\[
g = c \frac{\partial M_*}{\partial t}
\]

\( c \) = arbitrary constant and \( \frac{\partial M_*}{\partial t} \) is the star-formation rate.

So total change in stellar mass can be written as:

\[
\delta M_* = -\frac{1}{(1+c)} \delta M_g
\]

The leaky-box model

Can now derive the metallicity evolution, as before:

\[
Z(t) = Z_f \frac{M_*}{M_g}
\]

\[
\delta Z = \delta \left( \frac{M_*}{M_g} \right) = -\rho \frac{\delta M_g}{M_g}
\]

Now substitute for \( \frac{\partial M_g}{\partial t} \) from previous slide:

\[
\delta Z = -\rho \frac{\delta M_g}{(1+c) M_g}
\]

i.e., same as before, outflow simply reduces the yield from to an effective yield. Not really surprising that a leaky-box lowers final metallicity and exacerbates the G-dwarf problem.

The accreting-box model

Let's assume primordial gas is accreted at the star-formation rate, i.e., gas replacement.

As for the closed box model the production of metals per unit gas mass is the same, i.e.,

\[
\delta M_* = (Z - \rho) \delta M_g = (\rho - Z) \delta M_*
\]

However the conservation of mass is now:

\[
\delta M_g = -\delta M_* + f(t)
\]

In fact let's consider a replenishment model such that the gas mass stays constant:

\[
\delta Z = \frac{1}{M_g}[(\rho - Z) \delta M_* - Z \delta M_g] = \frac{1}{M_g} (\rho - Z) \delta M_*
\]

Closed box (as before) accreting box.
The accreting-box model

Integrating and assuming $Z_o=0$:

$$Z = \rho[1 - e^{-M_* / M_G}]$$

Assuming SF and infall have been progressing continuously i.e., $M_* \gg M_G$ then $Z \approx \rho$

And mass in stars which are more metal-poor than $Z$ is:

$$M_* (< Z) = -M_G \ln(1 - Z / \rho)$$

For the solar neighbourhood: $M_G \approx 10 M_\odot / \text{pc}^2$ and $M_* \approx 40 M_\odot / \text{pc}^2$ and for $Z \approx 0.7 Z_\odot$

Fraction of stars more metal-poor than $0.25 Z_\odot$ is $M(<0.25)/M(<0.7) \approx 10$

Closer to that observed than in the closed box model.

The G-dwarf problem

At least two possible solutions:

1) Disc formed from pre-enriched gas (left over from Bulge?).

2) Significant gas infall has occurred so that the remaining gas is NOT indicative of the amount of star-formation which has occurred.