Lecture 5: Surface brightness and the Sersic profile

- Surface brightness definition
- Surface brightness terminology
- The Sersic profile:
  - Some useful equations
  - Exponential profile
  - De Vaucouleurs profile
- Some examples:
  - Profiles
  - Equations

Surface brightness from AS1001

- Measure of flux concentration
- Units, mag/sq arcsec
- Range: 8-30 mag/sq/arsec
- High = compact = Low number!
- Low = diffuse = High number!
- Definition:
  \[
  \Sigma = I = \frac{f}{4\pi R^2}
  \]
  \[
  \mu \propto -2.5 \log_{10}(I) \quad [\text{analog. to } m \propto -2.5 \log_{10}(f)]
  \]
  \[
  \Rightarrow \mu \propto m + 5 \log_{10}(R)
  \]
  Distance independent in local Universe
Measuring light profiles

- NB, \( l(r) \) actually comes from an azimuthal average, i.e. of the light between isophotes of the same shape as the galaxy.
- NB we do not always see the steep rise in the core – if a galaxy has a small core, the atmospheric seeing may blur it out to an apparent size equal to the seeing (e.g. 1 arcsec).
- Often central profiles are artificially flattened this way.
- Need to model PSF.

Example of a galaxy light profile
The Sersic profile

- Empirically devised by Sersic (1963) as a good fitting fn

\[ I(r) = I_0 \exp \left( -\left( \frac{r}{\alpha} \right)^n \right) \]

- I(r) = intensity at radius r
- \( I_0 \) = central intensity (intensity at centre)
- \( \alpha \) = scalelength (radius at which intensity drops by e\(^{-1}\))
- \( n \) = Sersic index (shape parameter)

Can be used to describe most structures, e.g.,
- Elliptical: 1.5 < n < 20
- Bulge: 1.5 < n < 10
- Pseudo-bulge: 1 < n < 2
- Bar: n~0.5
- Disc: n~1

Total light profile = sum of components.

Sersic shapes
Connecting profile shape to total flux

The best method for measuring the flux of a galaxy is to measure its profile, fit a Sersic fn, and then derive the flux.

\[ L = \int_0^{2\pi} \int_0^\infty I(r) dr = 2\pi \int_0^\infty r \exp\left(-\left(\frac{r}{\alpha}\right)^n\right) dr \]

Substitute: \( x = \left(\frac{r}{\alpha}\right)^n \), \( r = x^n \alpha \), \( dr = nx^{n-1} \alpha \)

\[ L = 2\pi \int_0^\infty x^n \alpha^{n-1} \exp(-x) dx \]

\[ L = 2\pi \alpha^n \int_0^\infty x n^{-1} \exp(-x) dx \]

Recognise this as the Gamma fn (Euler's integral or factorial fn):

\[ \Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt = (z-1)! \]

So:

\[ L = 2\pi \alpha^n \Gamma(2n) \]

Or for integer \( n \):

\[ L = \pi \alpha^2 \Sigma (2n-1)! = \pi \alpha^2 (2n)! \]

Very important and useful formulae which connects the total luminosity to directly measureable structural parameters.

Most useful when expressed in magnitudes:

\[ 10^{-0.4m} = (2n)! \pi 10^{-0.4\mu} \alpha^2 \]

\[ -0.4m = -0.4\mu + \log[(2n)! \pi \alpha^2] \]

\[ m = \mu - 5 \log(\alpha) - 2.5 \log[(2n)! \pi] \]

- For \( n=1 \):
  \[ m = \mu_0 - 5 \log_{10} \alpha - 2 \]

- For \( n=4 \):
  \[ m = \mu_0 - 5 \log_{10} \alpha - 12.7 \]

- So for fixed \( m \) as \( \alpha \uparrow \mu_0 \uparrow I_o \)

- Note: SBs are like mags low value = high SB, high value=low SB !!
Q1) If a galaxy with a Sersic index of 1 has a measured SB of $\mu_o = 21.7$ mag/sq arcsec, a scale-length of 2'' and lies at a redshift of 0.1 what is its absolute magnitude? [Assume $H_o=100\text{km/s/Mpc}$.]

\[
m = \mu_o - 5 \log_{10} \alpha - 2
\]
\[
m = 21.7 - 1.5 - 2
\]
\[
m = 18.2 \text{mags}
\]
\[
z = \frac{v}{c}, v = H_o d \Rightarrow z = \frac{H_o d}{c}
\]
\[
d = \frac{cz}{H_o} = 300 \text{Mpc}
\]
\[
m - M = 5 \log_{10} d + 25
\]
\[
M = -19.2 \text{mags}
\]

Q2) $\mu_o$ for an elliptical with $n=4$ is typically 15 mag/sq arcsec at $m=19.7$ mags what $\alpha$ does this equate too?

\[
m = \mu_o - 5 \log_{10} \alpha - 12.7
\]
\[
\alpha = 10^{0.4[m - m - 12.7]}
\]
\[
\alpha = 3.3 \times 10^{-4}\text{''}
\]

Too small to measure therefore one often redefines the profile in terms of the half-light radius, $R_e$, also known as the effective radius.
• Scale-length to small to measure for high-n systems so adopt **half-light radius** ($R_e$) instead.

$$2\pi \int_0^\infty rI(r)dr = \frac{2\pi}{2} \int_0^\infty rI(r)dr$$

$2\gamma(2n,k) = \Gamma(2n) = 2\int_0^\infty r^{2n-1}e^{-r}dr = 2\int_0^\infty r^{2n-1}e^{-r}$, where $R_e = k^n\alpha$, or, $\alpha = \frac{R_e}{k^n}$

• Can now sub for $\alpha$ and rewrite Sersic profile as:

$$I(r) = I_e \exp\left(-\left[\frac{r}{(\frac{R_e}{k^n})^{\frac{1}{n}}}\right]^{\frac{1}{n}}\right) = I_e \exp\left(-k\left[\frac{r}{R_e}\right]^{\frac{1}{n}}\right)$$

• $k$ can be derived numerically for $n=4$ as $k=7.67$

$$I(r) = I_e \exp\left(-7.67\left[\frac{R}{R_e}\right]^{\frac{1}{4}}\right)$$

Therefore: $R_e = 3459\alpha$

From previous example: $\alpha = 3.3 \times 10^{-4}$''; $\alpha = 3.3 \times 10^{-4}$''

I.e., $R_e$ is measurable. $R_e = 1.15''$

Central SB is also very difficult to get right for ellipticals as its so high, much easier to measure the surface brightness at the half-light radius. I.e.,

$$I_e = I_e \exp(-7.67) = I_e 10^{-3.33}$$

$[\text{Re}] 10^{-x} = e^{\ln[10^{-x}]} = e^{-x\ln10}$

$$I(r) = I_e 10^{3.33}\left[\frac{R}{R_e}\right]^{\frac{1}{2}}$$

$$I(r) = I_e 10^{-3.33}\left[\frac{R}{R_e}\right]^{\frac{1}{2}} - 1]$$

$$I(r) = I_e \exp\{-7.67\left[\frac{R}{R_e}\right]^{\frac{1}{4}} - 1\}$$
This expression is known as the de Vaucouleur’s $r^{1/4}$ law and is commonly used to profile ellipticals galaxies. Note luminosity can be recast for $n=4$ as:

$$L = \pi I_o \alpha^2 (2n)!$$

For: $n = 4$, $\alpha = \frac{R_e}{3459}$, $I_0 = I_e 10^{3.33}$

$$\Rightarrow L = 7.2 \pi I_e R_e^2$$

Recently the more generalised form with $n$ free has become popular with galaxies having a range of $n$ from $10\rightarrow0.5$ (Note $n=0.5$ is a Guassian-like profile) as it fits the variety of structures we see:

$$I(r) = I_e \exp\{-k[(\frac{r}{R_e})^{1/n} - 1]\}$$

In reality profiles are a combination of bulge plus disc profiles with the bulges exhibiting a Sersic profile and the disc an exponential.

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Total light for 2 component system etc.

- including the bulge, we can describe the profile as the sum of a spiral and an elliptical power law:
  \[ \Sigma(R) = \Sigma_0^{(d)} \exp(-R/R_d) + \Sigma_e^{(b)} \exp(-7.67 ([R/R_e]^{1/4} - 1)) \]

- but if the bulge is rather flat (pseudo-bulge) we can just use a second n=1 exponential with a smaller radius:
  \[ \Sigma(R) = \Sigma_0^{(d)} \exp(-R/R_d) + \Sigma_0^{(b)} \exp(-R/R_b) \]

- for the disk-plus bulge version, we get the total luminosity
  \[ L = 2\pi \Sigma_0^{(d)} R_d^2 + 7.22 \pi R_e^2 \Sigma_e^{(b)} \]
Bulge to disc ratio

- a useful quantity to describe a spiral is the bulge-to-total light ratio, which we can find from the two luminosity terms

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Size of a galaxy?

- Measuring the size of a galaxy is non-trivial
  - Not clear where galaxy ends
  - Some truncate and some don’t
  - Need a standard reference
- By convention galaxy sizes are specified by the half-light radius ($R_e$) or by scale-length ($\alpha$)
- New quantitative classification scheme is based on the stellar mass versus half-light radius plane either for the total galaxy or for components….work in progress…